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Crossover of phase qubit dynamics in the presence of a negative-result weak measurement

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The coherent dynamics of a superconducting phase qubit is considered in the presence of both unitary evolution due to microwave driving and continuous nonunitary collapse due to a negative-result measurement. In the case of relatively weak driving, the qubit dynamics is dominated by the nonunitary evolution, and the qubit state tends to an asymptotically stable point on the Bloch sphere. This dynamics can be clearly distinguished from conventional decoherence by tracking the state purity and the measurement invariant. When the microwave driving strength exceeds a certain critical value, the dynamics changes to nondecaying oscillations: any initial state returns exactly to itself periodically in spite of the nonunitary dynamics. The predictions can be verified using a modification of a recent experiment.

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The problem of measurement of a single quantum system plays a fundamental role in our understanding of physical reality.[1] While the evolution of an isolated quantum system is governed by its Hamiltonian, the state evolution of a measured (open) quantum system arises from a nontrivial interplay between its internal Hamiltonian evolution and the "informational" evolution associated with a given measurement record.[2]

Experimental advances in the fabrication of superconducting and semiconductor qubits[3] provide unique possibilities to probe the quantum behavior of a single quantum system by weakly measuring it via mesoscopic detectors. For example, qubit evolution can be monitored by a weakly coupled quantum point contact or single-electron transistor, which behaves classically on the time scale defined by the qubit dynamics. The measurement record in this case is a fluctuating current[4] that is imperfectly correlated with the quantum state. Given the continuous measurement record, the qubit state is continuously collapsed due to quantum back action.[5,6]

Recently,[7] a variant of weak continuous measurement was demonstrated experimentally in which partial collapse is achieved by means of *registering no signal*. Realized with a superconducting "phase" qubit measured via tunneling,[8] it is the first solid-state demonstration of quantum null (negative-result) measurement effects proposed and discussed mainly in the context of quantum optics.[9–11] Contrary to naive expectation, the no-signal result leads to a change of the quantum state, providing a new type of qubit manipulation.

In this paper we consider the interplay of coherent dynamics of a phase qubit (see Fig. 1) due to unitary Schrödinger evolution (because of microwave driving) and due to continuous collapse under a negative-result measurement. We show the existence of a *critical value* for the ratio of the Rabi frequency Ω_R to the measurement rate Γ. For $2\Omega_R/\Gamma < 1$ the dynamics is dominated by the nonunitary evolution. The qubit state is continuously collapsed to a fixed asymptotic state, a special point on the Bloch sphere that depends on Ω_R/Γ but is independent of the initial conditions. Thus, any mixed state purifies. For $2\Omega_R/\Gamma > 1$, the dynamics changes qualitatively and shows *nondecaying* oscillations of the qubit state, so that no asymptotic state is reached and any initial state returns to itself after an oscillation period. The qubit does not completely purify: the purity undergoes nondecaying oscillations as well. Our results are valid only as long as no signal is measured by the detector; in the presence of microwaves the probability that the negative result persists decreases roughly exponentially in time.

We consider a phase qubit[7,8] which consists of a superconducting loop interrupted by a Josephson junction [Fig. 1(a)] and controlled by an external magnetic flux ϕ_{ext}. The qubit basis states $|0\rangle$ and $|1\rangle$ are the two lowest-energy states in the shallow "left" well [Fig. 1(b)] of the potential profile $V(\phi)$, where ϕ is the superconducting phase difference across the junction. A Rabi rotation of the qubit state is achieved by applying a resonant microwave signal $I_{\mu W}$. The measurement is performed by lowering the barrier (by changing the flux ϕ_{ext} that biases the qubit) for a finite time t. While the tunneling from the ground state $|0\rangle$ is still strongly suppressed, the excited state $|1\rangle$ may tunnel out, with a rate Γ, to the much deeper right well, where its energy relaxes rapidly [Fig. 1(b)]. This irreversible tunneling event can be detected by an inductively coupled SQUID, which switches to the finite-voltage state.[7,8] After one selects the cases of no tunneling, the resulting qubit state can be further analyzed by quantum state tomography.[7,12]

First we consider the case when no microwaves are ap-

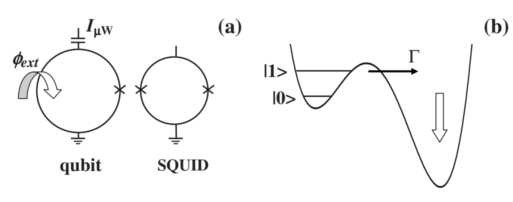

FIG. 1. (a) Schematic of a phase qubit controlled by a microwave current $I_{\mu W}$ and an external flux ϕ_{ext}, and inductively coupled to a superconducting quantum interference device (SQUID). (b) Lowest energy levels in the left well of the profile $V(\phi)$ represent the qubit states. Tunneling to the right well from state $|1\rangle$ is detected by the SQUID.

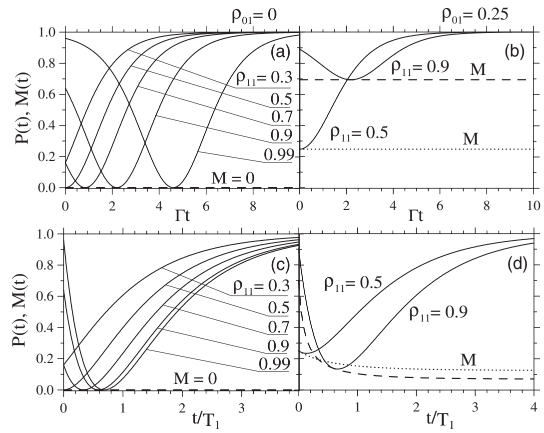

FIG. 2. Time dependence of purity P (solid lines) and murity M (dashed or dotted lines) in the absence of microwave driving for qubit state evolution due to (a), (b) negative-result measurement or (c), (d) zero-temperature energy relaxation. Notice the qualitative difference between the corresponding curves in the upper and lower panels for the same initial conditions [$\rho_{11}(0)$ is shown for each curve; $\rho_{01}(0)=0$ in (a), (c) and $\rho_{01}(0)=0.25$ in (b), (d)]. The relative scale of the upper and lower panels is chosen to maximize visual similarity of the curves in (a) and (c).

plied. Measuring the qubit for a sufficiently long time t, such that $\Gamma t \gg 1$, essentially results in a strong measurement: the qubit state is either collapsed onto state $|0\rangle$ (if no tunneling has happened) or destroyed (if tunneling has happened). However, measurement for a finite time $t \lesssim \Gamma^{-1}$ is weak: the qubit state is still destroyed if a tunneling event happens, but in the case of no tunneling, a negative result, the qubit density matrix ρ (in the basis of states $|0\rangle$ and $|1\rangle$) evolves continuously according to the quantum Bayes rule[5,7,10]

$$\rho_{00}(t) = \rho_{00}(0)/\mathcal{N}, \quad \rho_{11}(t) = \rho_{11}(0)e^{-\Gamma t}/\mathcal{N}, \tag{1}$$

$$\rho_{01}(t) = \rho_{01}(0)\sqrt{\rho_{00}(t)\rho_{11}(t)/[\rho_{00}(0)\rho_{11}(0)]}e^{i\varphi}, \tag{2}$$

where $\mathcal{N} \equiv \rho_{00}(0)+\rho_{11}(0)e^{-\Gamma t}$. Note that Eqs. (1) and (2) describe the ideal change in ρ in the rotating frame,[2] which cancels the energy difference of states $|0\rangle$ and $|1\rangle$. The qubit acquires a known phase φ due to a small shift of the level spacing under the change of ϕ_{ext};[7] in what follows we neglect this effect, assuming $\varphi=0$. In Eqs. (1) and (2) we have neglected the decoherence due to the environment since experimentally it can be made much slower than the tunneling rate Γ.[7]

It proves convenient (Fig. 2) to characterize the quality of the qubit state by the purity $P \equiv 2\,\mathrm{Tr}\,\hat{\rho}^2 - 1 = x^2+y^2+z^2$, which is an invariant of unitary transformations; here $x = 2\,\mathrm{Re}\,\rho_{01}$, $y=2\,\mathrm{Im}\,\rho_{01}$, and $z=\rho_{00}-\rho_{11}$ are Bloch components. (Note that the linear entropy $1-P$ is a one-to-one function of the von Neumann entropy $S=-\mathrm{Tr}\hat{\rho}\log_2\hat{\rho}$, satisfying $1-P \leqslant S$.) Another important state characteristic is the "murity" $M=|\rho_{01}|^2/(\rho_{00}\rho_{11})=(x^2+y^2)/(1-z^2)$,[5,13] which is an invariant of the measurement evolution [see Figs. 2(a) and 2(b)]. Obviously, $P=M=1$ for a pure state, and it is easy to show that $P \geqslant M$ always.

From Eqs. (1) and (2) one obtains the purity evolution due to negative-result measurement: $P(t)=P(0)+[1-P(0)](1-e^{-\Gamma t}/\mathcal{N}^2)$. An initially pure state remains pure, while an initially mixed state tends to the pure state $|0\rangle$ asymptotically, for $t \gg 1/\Gamma$. However, the purity of an initially mixed state increases monotonically in time only if $\rho_{11}(0) \leqslant 1/2$ [see Figs. 2(a) and 2(b)]. In contrast, for $\rho_{11}(0) > 1/2$ the state first becomes more mixed, reaching minimal purity $P_{min}^{meas}=M(0)$ at a time

$$\tau_{min}^{meas} = (1/\Gamma)\ln\{\rho_{11}(0)/[1-\rho_{11}(0)]\} \tag{3}$$

when $\rho_{00}(t)=\rho_{11}(t)=1/2$, and only then starts to purify.

A qualitative explanation of the nonmonotonic behavior of $P(t)$ is purely classical and based on the informational character of the evolution due to negative-result measurement. Consider a (classical) qubit state which is known to be more likely in state $|1\rangle$ than state $|0\rangle$. If the qubit does not tunnel out after a small time t, then the likelihood that the qubit is in the nondecaying state $|0\rangle$ slightly increases. Thus, the uncertainty (entropy) of the qubit state increases, and so the purity $P(t)$ decreases. If the qubit still has not decayed after a sufficiently long time, we are practically sure that the state is $|0\rangle$, so the entropy decreases and purity increases. Notice that Eq. (3) does not depend on $\rho_{01}(0)$, thus allowing a purely classical interpretation.

An important question is whether or not the qubit evolution (1) and (2) due to negative-result measurement can be imitated by the evolution due to conventional decoherence characterized by the energy relaxation time T_1 and the dephasing time T_2.[14] As we show below, the answer is no; the two evolutions are significantly different.

Let us start the comparison assuming zero-temperature relaxation and minimal dephasing rate $T_2=2T_1$. In the case $\rho_{01}(0) \neq 0$, the most obvious difference between the two kinds of evolution is the behavior of the murity $M(t)$ [see Figs. 2(b) and 2(d)]. The murity is a constant for the measurement evolution, while in the case of decoherence $M(t)$ decreases to a value $M(0)\rho_{00}(0)$, even though the qubit approaches the same ground state. If $\rho_{11}(0) > 1/2$, then the minimal purity in the decoherence scenario, $P_{min}^{T_1} = 1-[1-|\rho_{01}(0)|^2/\rho_{11}(0)]^2$, is also smaller than $P_{min}^{meas}=M(0)$. This minimum is reached at a time

$$\tau_{min}^{T_1} = T_1\ln\{2\rho_{11}(0)/[1-|\rho_{01}(0)|^2/\rho_{11}(0)]\} \tag{4}$$

that is much less sensitive than τ_{min}^{meas} [Eq. (3)] to the initial conditions. In particular, $\tau_{min}^{T_1}$ approaches the finite value $T_1 \ln 2$ for $\rho_{11}(0) \to 1$, while τ_{min}^{meas} grows logarithmically. Notice [Figs. 2(b) and 2(d)] that in both evolutions the curve $P(t)$ touches the curve $M(t)$; in the measurement case this happens at τ_{min}^{meas} while in the decoherence case this happens at $t=T_1 \ln 2\rho_{11}(0) < \tau_{min}^{T_1}$ when $P(t)=M(t)=2M(0)\rho_{00}(0)$. If $\rho_{01}(0)=0$, Eqs. (3) and (4) still imply a difference in their sensitivity to initial conditions that is evident in Figs. 2(a) and 2(c).

When dephasing exceeds its minimal value ($T_2 < 2T_1$), the asymptotic murity $M(\infty)$ in the decoherence scenario always tends to zero, thus making the two evolutions still more distinct. A finite temperature leads to a similar effect, and also makes the asymptotic qubit state different from the asymptotic state $|0\rangle$ of the evolution due to negative-result measurement. Thus, the two evolutions are always significantly different. While we have considered only the Markovian model of decoherence characterized by T_1 and T_2, we believe that our measurement process (1) and (2) cannot be imitated by any model of decoherence. This is because an initially pure state generally becomes mixed due to decoherence (at least temporarily), while in the process of measurement it remains pure.

Now let us consider the state dynamics due to negative-result measurement in the presence of microwave driving exactly at resonance. Differentiating Eqs. (1) and (2) over time and adding the evolution due to Rabi oscillations, we obtain the following evolution in the rotating frame:

$$\dot{\rho}_{00} = -\dot{\rho}_{11} = -\Omega_R \,\mathrm{Im}\,\rho_{01} + \Gamma \rho_{00}\rho_{11}, \tag{5}$$

$$\dot{\rho}_{01} = i\frac{\Omega_R}{2}(\rho_{00}-\rho_{11}) - \frac{\Gamma}{2}(\rho_{00}-\rho_{11})\rho_{01}. \tag{6}$$

Here we consider driving that shows up as a σ_x term in the Hamiltonian $\mathcal{H}=(\Omega_R/2)(|0\rangle\langle 1|+|1\rangle\langle 0|)$; σ_y evolution can be easily incorporated via a finite rotation (for simplicity we assume the absence of a σ_z term). Although Eqs. (5) and (6) are deterministic, their nonlinear terms resemble those in the case of noisy weak measurement.[5]

The solution is conveniently expressed in terms of Bloch components. The evolution of x decouples, and setting $h \equiv 2\Omega_R/\Gamma$ and $\omega \equiv (\Gamma/2)\sqrt{1-h^2}$, we obtain

$$x(t) = -x(0)[1-h^2]/\mathcal{D}(t), \tag{7}$$

$$y(t) = \{h - y(0) - h[1-hy(0)]\cosh \omega t - h\sqrt{1-h^2}z(0)\sinh \omega t\}/\mathcal{D}(t), \tag{8}$$

$$z(t) = \{(1-h^2)z(0)\cosh \omega t + \sqrt{1-h^2}[1-hy(0)]\sinh \omega t\}/\mathcal{D}(t), \tag{9}$$

where

$$\mathcal{D}(t) \equiv h[h-y(0)] - [1-hy(0)]\cosh \omega t - \sqrt{1-h^2}z(0)\sinh \omega t.$$

The most important observation is a critical point at $h=1$. Below the critical value, $h<1$, the evolution is dominated by the measurement, and the qubit state asymptotically collapses to a stable value on the Bloch sphere with coordinates $x_\infty=0$, $y_\infty=h$, $z_\infty=\sqrt{1-h^2}$. This occurs independently of the initial conditions. The asymptotic state $r_{as}\equiv\{x_\infty,y_\infty,z_\infty\}$ attracts the trajectories on the Bloch sphere [see Fig. 3(a)], while the state $r_{rp}\equiv\{x_\infty,y_\infty,-z_\infty\}$ repels trajectories. It is simplest to visualize the dynamics starting from a point on the great circle $y^2+z^2=1$. Then the presence of microwaves rotates the state around the circle in a clockwise direction (when viewed from the positive x axis), while the measurement evolution rotates it either clockwise or counterclockwise toward state $|0\rangle$ (north pole). At the points r_{as} and r_{rp}, the two rotations exactly compensate each other, creating the stable and unstable equilibrium states.

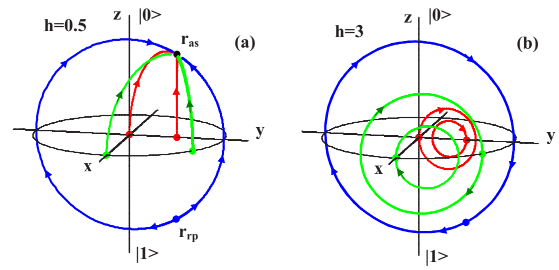

FIG. 3. (Color online) (a) Trajectories of the qubit state evolution in Bloch coordinates for $h\equiv 2\Omega_R/\Gamma=0.5$, starting from several initial states. All states purify and approach the attractive asymptotic point r_{as}, while r_{rp} is the repulsive point. (b) Evolution for $h=3$ from the same initial states: nondecaying oscillations with period $4\pi/\Gamma\sqrt{h^2-1}$.

At the critical value $h=1$, the equilibrium states r_{as} and r_{rp} coincide, and the asymptote is achieved not exponentially, but in a power-law fashion: $z(t)\simeq 4/(\Gamma t)$, while $y(t)\simeq 1-8/(\Gamma t)^2$. Above the critical value, $h>1$, the state does not stabilize at all, and the qubit performs *nondecaying oscillations* with period $T_{osc}=4\pi/\Gamma\sqrt{h^2-1}$ [see Figs. 3(b) and 4]. This means that every state returns exactly to itself after T_{osc}, in spite of the nonunitary dynamics altering pure Rabi oscillations,[15] thus being an example of a (nonoptimal) quantum undemolition measurement.[13] Notice that the oscillating evolution is symmetric about the equatorial plane even though the negative result drives the system toward the north pole.

The state purity evolves as $\dot{P}=\Gamma z(1-P)$, so a pure state remains pure. For $h<1$ a mixed state eventually purifies, and asymptotically $P\simeq 1-e^{-z_\infty \Gamma t}$. Also, $M(t)$ eventually approaches 1, though at a later time than the purity ($dM/dt\to 0$ for $h\to 0$). For $h>1$ the purity and murity oscillate because of the state oscillation, so a mixed state does not purify completely. In the case when h is only a little over 1, a mixed state purifies almost completely, but the purity still returns to its initial value after a long period.

Let us discuss now how, in the presence of microwaves, negative-result measurement evolution differs from decoher-

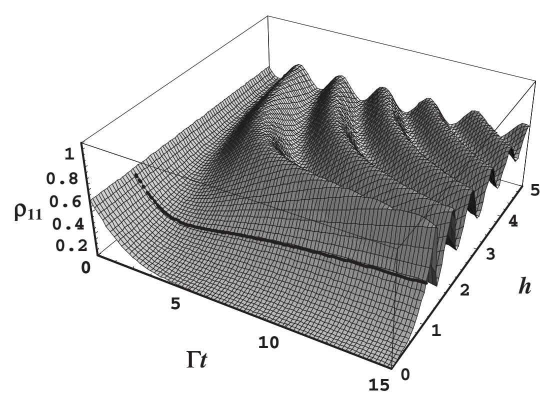

FIG. 4. Population of the excited state ρ_{11} vs scaled Rabi frequency h and time t for a totally mixed initial state. Crossover from nonoscillatory to undamped oscillatory dynamics occurs at $h=1$ (thick line). Decoherence is neglected.

ence evolution. In the decoherence case, the measurement terms in Eqs. (5) and (6) should be replaced by $-\gamma_1(\rho_{00}-p_{st})$ and $-\gamma_2\rho_{01}$ respectively,[14] where $\gamma_{1,2}\equiv 1/T_{1,2}$ and p_{st} is the equilibrium ground state population (in experiment[7,8] $p_{st}\approx 1$). Introducing $\tilde{h}\equiv 2\Omega_R/\gamma_1$ and $d\equiv\gamma_2/\gamma_1=T_1/T_2\geqslant 1/2$, we find

$$x(t)=x(0)\exp(-\gamma_2 t), \tag{10}$$

$$y(t)=y_a+\exp[-t(\gamma_1+\gamma_2)/2]\{c_y\cosh\tilde{\omega}t + (\gamma_1/2\tilde{\omega})[\tilde{h}c_z+c_y(1-d)]\sinh\tilde{\omega}t\}, \tag{11}$$

$$z(t)=z_a+\exp[-t(\gamma_1+\gamma_2)/2]\{c_z\cosh\tilde{\omega}t - (\gamma_1/2\tilde{\omega})[\tilde{h}c_y+c_z(1-d)]\sinh\tilde{\omega}t\}, \tag{12}$$

where $\tilde{\omega}\equiv(\gamma_1/2)[(1-d)^2-\tilde{h}^2]^{1/2}$, $c_z\equiv z(0)-z_a$, $c_y\equiv y(0)-y_a$, and $x_a=0$, $y_a=2(2p_{st}-1)\tilde{h}/(\tilde{h}^2+4d)$, $z_a=4(2p_{st}-1)d/(\tilde{h}^2+4d)$ are the asymptotic values.

The evolution (10)–(12) resembles that of a damped oscillator and is quite different from the evolution (7)–(9). For $\tilde{h}<|1-d|$ the overdamped regime is realized (no oscillations), while for $\tilde{h}>|1-d|$ we have damped oscillations (underdamped regime). For arbitrary $\tilde{h}$ the Bloch components approach x_a, y_a, and z_a, and the asymptotic purity is $P_a=4(2p_{st}-1)^2(\tilde{h}^2+4d^2)/(\tilde{h}^2+4d)^2$, implying that a mixed state never becomes pure except at zero temperature ($p_{st}=1$) in the absence of microwaves. Even an initially pure state becomes mixed, with asymptotic purity and murity both close to zero for large $\tilde{h}$. We can conclude that in the presence of microwaves the qubit dynamics due to decoherence is still qualitatively different from the dynamics due to negative-result measurement.

The phase qubit evolution due to negative-result measurement discussed in this paper can be verified experimentally using quantum state tomography[12,16] in the same way as in the recent experiment of Ref. 7, which has verified the evolution (1) and (2). In a realistic situation, the decoherence evolution is always added to the measurement evolution; however, as we discussed above, the qualitative effects of the two evolutions are easily distinguishable. Moreover, the decoherence can be made more than ten times slower than the evolution due to measurement,[7] justifying neglect of decoherence in our analysis of the crossover from asymptotic qubit purification to nondecaying oscillations.

Notice that the predicted qubit evolution due to negative-result measurement can be seen experimentally only as long as the qubit has not decayed. The probability that the qubit has not decayed by time t is $\mathcal{P}(t)=\exp[-\Gamma\int_0^t dt'\rho_{11}(t')]$. In the absence of microwaves ($h=0$), it becomes $\mathcal{P}(t)=\rho_{00}(0)+\rho_{11}(0)e^{-\Gamma t}$ and remains finite with increasing time. However, with added microwaves ($h\neq 0$), $\mathcal{P}(t)$ tends to zero at $t\to\infty$, which means that the qubit eventually decays. Since predictions requiring unreasonably small values of $\mathcal{P}$ are hardly accessible experimentally, we have checked that the qualitative picture of our results can still be seen at the cutoff level $\mathcal{P}>5\%$. While the close vicinity of the critical point ($h=1$) is the hardest regime for experimental analysis, the predicted nonoscillatory evolution at $h<1$ as well as a few nondecaying oscillations at $h\gtrsim 3$ should be observable experimentally with a minor modification of the experiment.[7]

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